BEAM DYNAMICS IN STRIPLINE LINEAR INDUCTION ACCELERATORS

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I. INTRODUCTION

Stripline (parallel plate transmission line) pulsed power modules have been considered for application to advanced high current linear accelerators.[1] Some advantages of the stripline designs include compact size, easy maintenance, and most importantly, the small number of switches required (one switch per 2 MeV). The principle drawback of stripline designs is that they impart a net transverse force to particles in the gap. This is shown to result in randomized transverse momentum, and net, constructive transverse guiding center motion.

In this paper, a semi-quantitative analysis of several facets of the problem is presented. Section II deals with the zero-order orbital mechanics of particle motion in the accelerator. The important result is that net guiding center motion is more important than temperature increase. In Section III, the magnitude of the asymmetric magnetic field, which drives the beam off axis, is calculated in an approximate fashion for several stripline configurations. In Section IV, the overall importance of the problem, and the beneficial results of modifications to the stripline design are presented.

II. RELEVANT ASPECTS OF PARTICLE MOTION

Several aspects of single particle dynamics are relevant to this problem. for a narrow accelerating gap of length ℓ, the change in transverse momentum $\delta p_\perp$ is given by

$$\delta \underset{\sim}{p}_\perp \approx \frac{\ell}{c} \ e(\underset{\sim}{E}_\perp + \underset{\sim}{v} \times \underset{\sim}{B}_\perp) \qquad (1)$$

$E_\perp$ and $B_\perp$ are the transverse electric and magnetic fields, and v is the particle velocity. The criterion for a gap to be narrow is that $\omega_c \ell/v < 1$, $\omega_c = eB_z/\gamma m$, $\gamma = (1 - v^2/c^2)^{-1/2}$.

For typical high current linac parameters of B_z = 15 kG, γ = 20, ℓ = 5 cm, we have $\omega_c \ell/v \approx 2.2$.

If $\omega_c \ell/v > 1$, the $\delta p_\perp$ of Equation (1) is an upper bound. Particles undergo what is effectively a drift motion similar to that of Ref. 2. The importance of beam transverse motion is that it may cause loss of beam particles due to the transverse displacement, and also that it may result in an increase in beam temperature.

The growth of beam transverse momentum is governed by the cyclotron motion between accelerating gaps spaced a distance L apart. We can show in general that the transverse momentum of a particle leaving the (n + 1)th gap is related to that leaving the nth gap by

$$\left| p_{\perp n+1} \right|^2 = \left| p_{\perp n} \right|^2 + (\delta p)^2 + \delta p[p_{nx} \cos\theta + p_{ny} \sin\theta]$$

where

$$\theta = (\omega_c L)/c \ .$$

In the cases of interest, $\theta \gg \pi$, so that in an "ensemble" average over a distribution of θ values,

$$\langle p_{\perp n+1}^2 \rangle \ = \ \langle p_{\perp n}^2 \rangle + \delta p^2$$

or

$$\langle p_{\perp n}^2 \rangle \ \approx \ n \delta p^2$$

The resulting transverse velocity is

$$[\langle \beta_\perp^2 \rangle]^{1/2} = p_\perp / (\sqrt{n} \ p_\parallel) \qquad (2)$$

where $p_\parallel \approx \gamma mc$ is the axial particle momentum. The slow transverse momentum growth implied by (2) is in contrast to the image displacement instability. The reason is that in image displacement,[2] large transverse displacements result in larger transverse motions. For large enough transverse forces, the amplification overcomes cancellation. The net displacement which results due to $p_\perp$ is given by the larmor radius $r_L \sim \frac{p_\perp}{\gamma m \omega_c}$ and increases as $\sqrt{n}$.

The distance of a particle from its guiding center is[3]

$$\underset{\sim}{R} \ = \ \frac{\underset{\sim}{p} \times \hat{z}}{eB_z} \qquad (3)$$

After traversing a gap, the displacement from the new guiding center is

$$\underset{\sim}{R}^1 = \frac{(\underset{\sim}{p} + \delta \underset{\sim}{p}_\perp) \times \hat{z}}{eB_z} \qquad (4)$$

Under the assumption of a narrow gap, the particle position does not change through the gap. Thus, the guiding center position ξ changes by $\underset{\sim}{R}^1 - \underset{\sim}{R}$, or

$$\delta \underset{\sim}{\xi} \ \approx \ \frac{\delta \underset{\sim}{p} \times \hat{z}}{eB_z} \qquad (5)$$

which implies

$$\xi \ \approx \ \frac{n \ \delta p \cdot z}{B_z} \qquad (6)$$

In terms of field components, Equation (6) gives the result,

$$\xi \ \approx \ \frac{n \ell B_\perp}{B_z} \qquad (7)$$

indicating that particles follow the magnetic field lines in the gap. It is interesting to note that this process has some similarity to collisional plasma diffusion across magnetic field lines.[4] The growth in guiding center displacement is proportional to the number of gaps n, so this represents the most serious off-center motion. In Sec. III, we evaluate $B_\perp$ for various stripline designs.

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14. ABSTRACT

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III. THE ASYMMETRIC MAGNETIC FIELD

The simplest relevant geometric configuration is the parallel plate transmission line with an accelerating gap shown in Figure 1. For a beam load of current I, traversing a gap of voltage $V = E_z\ell$, the transverse magnetic field is $\mu_0 I/d$ (d = parallel plate width). As a result, we find in Mks units,

$$\frac{\delta p_\perp}{\delta p_z} = \frac{377 I}{V} \frac{\ell}{d} \tag{8}$$

This simple model ignores the spatial inhomogeneity of the current flow, and magnetic fields outside the plates.
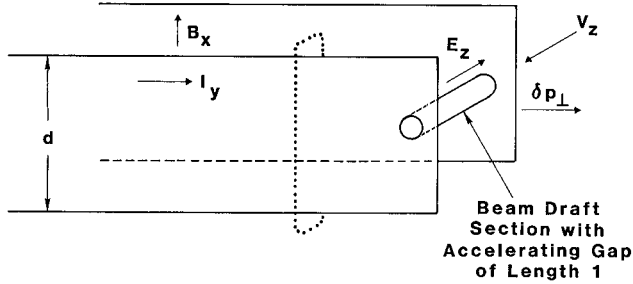

Figure 1. Parallel plate transmission line configuration-- the dotted line shows an idealized line integral path.

A key point illustrated by the dashed line in Figure 1 is that the transverse field will always be present because the line integral will always enclose current. A convenient approach which allows semi-quantitative analysis is to break transmission line segments into square dipole loops as shown in Figure 2.
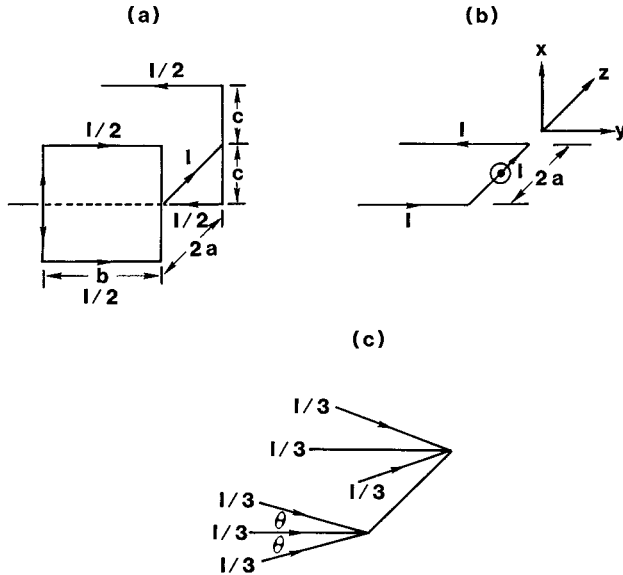

Figure 2. Idealizations of the current flow in parallel plate lines. (a) A line with split current flow to reduce B_x, (b) A plane dipole loop showing vector directions and relevant lengths, (c) Idealized converging flow in a stripline.

It is straightforward to show that the vector potential for a square current loop at $z = \mp a$, and $y = 0$, $-b$ is given by

$$\frac{4\pi \tilde{A}}{\mu_0 I} = \hat{z} \ln \frac{a - z + [(z - a)^2 + x^2 + y^2]^{1/2}}{-a - z + [(z + a)^2 + x^2 + y^2]}$$

$$- \hat{z} \ln \frac{a - z + [(z - a)^2 + x^2 + (y + b)^2]^{1/2}}{-a - z + [(z + a)^2 + x^2 + (y + b)^2]^{1/2}}$$

$$+ \hat{y} \ln \frac{-y + [y^2 + x^2 + (z + a)^2]^{1/2}}{-y - b + [(y + b)^2 + x^2 + (z + a)^2]^{1/2}}$$

$$- \hat{y} \ln \frac{-y + [y^2 + x^2 + (z - a)^2]^{1/2}}{-y - b + [(y + b)^2 + x^2 + (z - a)^2]^{1/2}} \tag{9}$$

Equation (9) may be applied in a number of interesting ways. The simplest is to find B_x at $z = x = y = 0$. This is given by:

$$B_x = \frac{\mu_0 I (a^2 + b^2)^{1/2}}{2\pi ab} \tag{10}$$

The $b \to \infty$ limit of Equation (10) gives a magnetic field of half the magnitude expected for current elements extending to $\pm \infty$. This illustrates the effect of the finite extent of current flow in the y direction on the total magnetic field.

A comparison between the parallel plate and the current loop results is of interest. When we include a factor of two correction for finite current flow, we have

$$\frac{B_x \text{ plate}}{B_x \text{ loop}} = \frac{\pi a}{c} ,$$

in the limit $b \to \infty$.

We may illustrate the importance of the transverse dislacement by inserting typical parameters[1] into Equations 7 and 10. Given a = 20 cm, ℓ = 5 cm, I = 5×10^4 A, B_z = .75 T, 4 accelerating gaps, and b $\approx$ 50 cm, we have B_x = 490 Gauss, ξ = 2.6 cm and $\langle p_\perp^2 \rangle^{1/2}/p_\parallel \approx .07$. The transverse displacement of approximately 3 mm per accelerating gap is unacceptable for for most applications.

A more accurate approximation than the parallel plate or single loop is the "triple loop" of Figure 2c which consists of three converging dipole loops each carrying a current I/3. The resulting transverse field is $B_x = (\mu I(a^2 + b^2)^{1/2}/6\pi ab) \cdot (1 + 2\cos\theta)$, indicating that a converging flow reduces B_x by $\approx (1 + 2 \cos\theta)/3$. A logical extension of this approach is to divide the flow into n small segments distributed evenly in θ, or $\cos\theta$. For current distributed uniformly in θ, we find

$$B_x \approx \frac{\mu_0 I(a^2 + b^2)^{1/2}}{2\pi ab} (\sin\theta/\theta) \tag{11}$$

This same approach allows us to find the effect of a slot in a parallel plate line which subtends the angle $\mp\theta_1$, in a line of convergence angle $\mp\theta_2$, as shown in Figure 3.
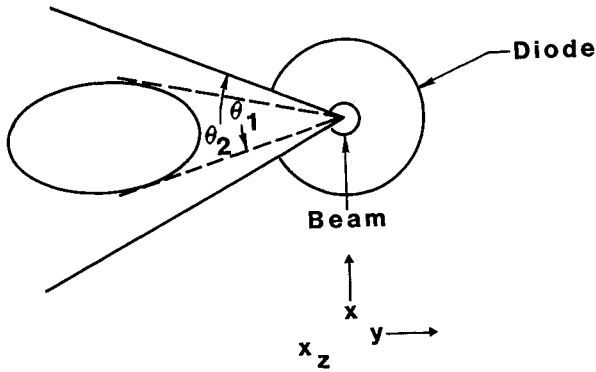
Figure 3. An idealized slotted line feeding a diode with θ_1 as the slot half angle and θ_2 as the line half angle.

For the case of the slot, the transverse field is found to be

$$B_x = \left[\frac{\mu_0 I(a^2 + b^2)^{1/2}}{2\pi ab}\right] \left[\frac{\sin \theta_2 - \sin \theta_1}{\theta_2 - \theta_1}\right] \quad (12)$$

The significance of Equation (12) may be assessed by using reasonable values for a slot, for example $\theta_1 = 35°$, and $\theta_2 = 55°$, resulting in a reduction of $\approx$ 20% in B_x. Thus, slots in the transmission line are not expected to reduce transverse motion significantly.

Note that for the cases of Equations (11) and (12), the meaning of the dimension "b" is nebulous. It can be taken as the distance from the end of the constant impedance transmission line to the beam.

IV. METHODS OF CORRECTING OR REDUCING THE PROBLEM

A number of methods have been suggested to reduce the value of B_x, in addition to the slot. The most obvious is delineated by Figure 2a. The transmission line is directed along the x-axis in the vicinity of the beam to some location $x = \mp c$, is split from $y = 0$, $x \mp c$ to $y = -b$, $x = \mp c$, and reconnects at $y = -b$, $x = 0$. The magnetic field for this configuration is

$$B_x = \frac{\mu_0 I ab}{2\pi (x^2 + a^2 + b^2)^{1/2}} \left(\frac{1}{x^2 + a^2}\right)$$
$$+ \frac{\mu_0 I}{2\pi a} \left(1 - \frac{b}{(a^2 + b^2)^{1/2}}\right) , x = c \quad (13)$$

The "source" region for each of the contributions in Equation 13 may be given as follows: $1/(x^2 + a^2)$ is from $-b + \varepsilon > y > 0$, and the last term originates from $y < -b - \varepsilon$. The zero order behavior of B_x is (taking $b \to \infty$),

$$B_x = \frac{\mu_0 I}{2\pi (x^2 + a^2)} ,$$

or $\approx a^2/(x^2 + a^2)$ lower than the value for the plane loop of Equation (10). The key to reducing the transverse field is then to maximize x/a, the distance between the two split lines. For example, if $x/a = 2$, B_x is reduced to ~20% of its original value.

A better idea of the various trade-offs inherent in Equation (13) is found in Figures 4 and 5. An example of a reasonable operating point is $x/a = b/a = 2$ resulting in $B_x = .25\ \mu_0 I/2\pi a$. More extreme geometries can result in even lower transverse fields. The limit $b \gg a$, is shown in Figure 6. If large values of stripline separations are allowed, very small fields (30 G or less) will result.
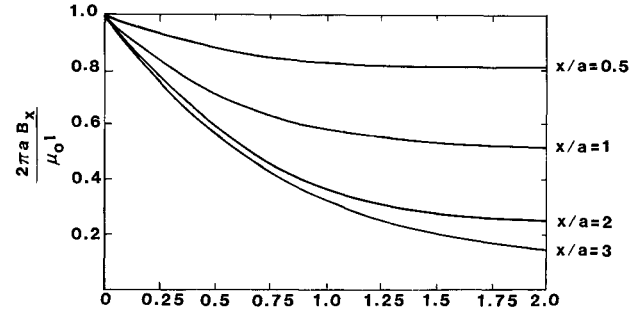

Figure 4. Normalized transverse magnetic field as a function of the length of the split section (b).
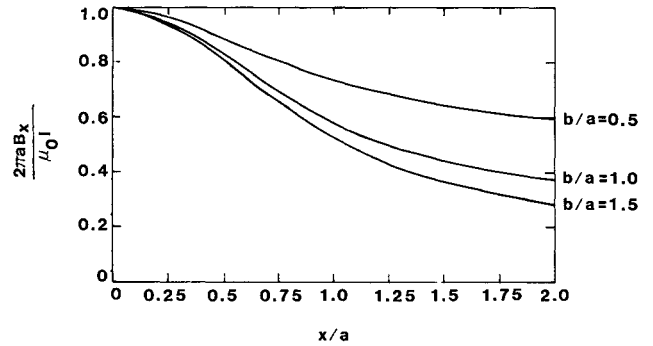

Figure 5. Normalized transverse magnetic field as a function of the distance from the split section to the beam pipe.
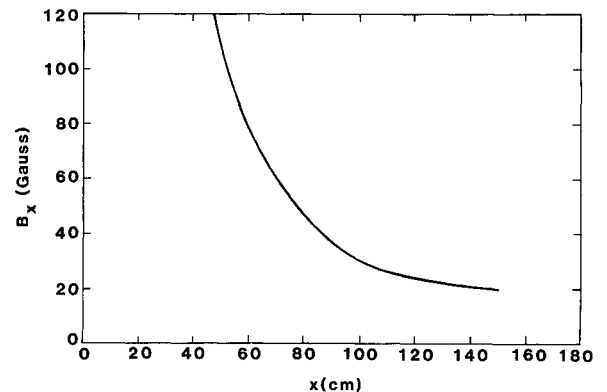

Figure 6. B_x for large b, a = 15 cm, and 50 kA beam current.

A second method of correcting the transverse "kick" was suggested by M. Buttram. If a uniform transverse field is produced by field coils or permanent magnets in each gap, the B_x field due to the asymmetric feed can be cancelled. Based on the discussion of Section I, the compensating coils do not necessarily need to be placed in the accelerating gap because the net guiding center motion is proportional to the total $\int B_x d\ell$.

The main disadvantage of this method is that the compensating field is only appropriate for one value of the beam current I. The beam would certainly be deflected during the current rise and fall.

Another method of suppressing the transverse impulse was suggested by the author and is depicted in Figure 7. If the gap is slanted at an angle θ such that $\tan \theta = B_x/E_z$, there is no net transverse force. It is apparent from the preceding relation that this will work only if the impedance of the accelerator as a whole is fixed as a function of time. Due to the approximately fixed impedance characteristics of foilless diodes, this is possible, at least in principle.
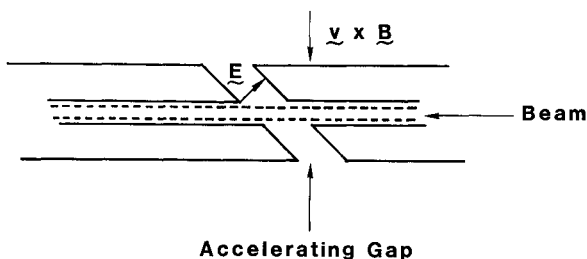

Accelerating Gap

Figure 7. A slanted accelerating gap in which $\underset{\sim}{E}$ compensates for $(\underset{\sim}{v} \times \underset{\sim}{B})_\perp$.

V. CONCLUSION

It is clear from the discussion of Section II that the transverse displacements for a simple stripline configuration are prohibitive and cannot be corrected by a small slot. The most favorable option for suppressing the problem is the use of lines split by a distance $2x \gg 2a$. If this is not feasible, or further reduction in B_x is necessary, then several methods are available to externally reduce the problem, at least for fixed impedance.

An important question which must still be addressed is the manner in which these transverse impulses couple to beam instabilities.

Finally, note that a number of pulse reflection, displacement current and transit time effects have been ignored. This is justified by noting that after the pulse rise time, only the net current (diode current) contributes to the net magnetic field. Pulse reflection concepts only provide a "shorthand" for demonstrating this fact. Nevertheless, the approach used here is strictly accurate only for the current "flattop", and the definition of "b" is somewhat nebulous until a time $b/c\sqrt{\varepsilon}$ into the pulse.

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